$$E_x = A\sqrt{z}J_p(kz) + B\sqrt{z}J_{-p}(kz)$$

where

$$p = [(z_c/\bar{\lambda})^2 + 1/4]^{1/2} \geq 1/2.$$

Reference to the asymptotic expression[6]

$$\sqrt{z}J_{-p}(kz) \approx z^{1/2-p},$$

which holds for $z \rightarrow 0$, shows that this part of the solution must be rejected to obtain a finite solution at the origin, corresponding to the case in which the plasma is backed by a metal wall as in Fig. 2(b).[7] The solution for the plasma with the inverse quadratic density distribution is then

$$E_x = \sqrt{z}J_p(kz)$$

where $p \approx z_c/\bar{\lambda}$ (assuming $z_c > \bar{\lambda}$).

Now, comparing (6) and (10) (with $\theta = 0$) we see that the waveguide and plasma shown in Fig. 2 exhibit analogous behavior with respect to a TE wave propagating toward the origin. In particular, the wave undergoes "reflection" in the neighborhood of the critical density z_c or the critical cross section r_c. Similarly, an evanescent wave arises at the critical point and damps out toward the origin. Thus, analog experiments carried out with appropriate tapered-waveguide configurations appear to be useful for simulating inhomogeneous plasma media in the vicinity of critical densities. Such experiments would require slowly-varying taper structures in order to minimize effects due to generation of spurious modes at junctions with other microwave elements.

HERBERT LASHINSKY
Plasma Physics Lab.
Princeton University
Princeton, N. J.

[7] L. S. Taylor, "Reflection of a TE wave from an inverse parabolic ionization density," IRE TRANS. ON ANTENNAS AND PROPAGATION (*Correspondence*), vol. AP-9, pp. 582–583; November, 1961.

The Diffraction Loss Curve for Nonconfocal Spherical Mirrors

The diffraction loss curve vs Fresnel number for confocal spherical mirrors was shown by Fox and Li[1] and by Goubau and Schwering.[2] Boyd and Gordon[3] suggested a possibility of applying the above theory to a nonconfocal mirror system, by assuming the diffraction loss to be equal to that of an equivalent confocal system having the same spot size. They proposed to express the diffraction loss in term of the parameter

$$\frac{a'^2}{d\lambda}\left[2\frac{d}{b'} - \left(\frac{d}{b'}\right)^2\right]^{1/2}$$

in place of the conventional Fresnel number N.

The approximation by Boyd and Gordon has been found nearly valid by Fox and Li[4] for the range of $0.2b' < d < 1.8b'$ in the calculation of infinite strip curved mirror interferometers for $N = 0.5$.

Using the Boyd and Gordon approximation, the diffraction loss for a nonconfocal system may be expressed as a function of the above parameter, which can be modified to the form

$$N\left[2\left(\frac{b'}{d}\right) - 1\right]^{1/2}$$

where

b' = radius of curvature of mirrors
d = spacing between mirrors
N = Fresnel number
$= a'^2/b'\lambda$
a' = radius of mirrors.

The above new parameter corresponds to the Fresnel number N for a confocal system, and the diffraction loss for a nonconfocal system can be easily obtained using the loss curve for a confocal system by replacing N to the form modified by the factor

$$\left[2\left(\frac{b'}{d}\right) - 1\right]^{1/2}.$$

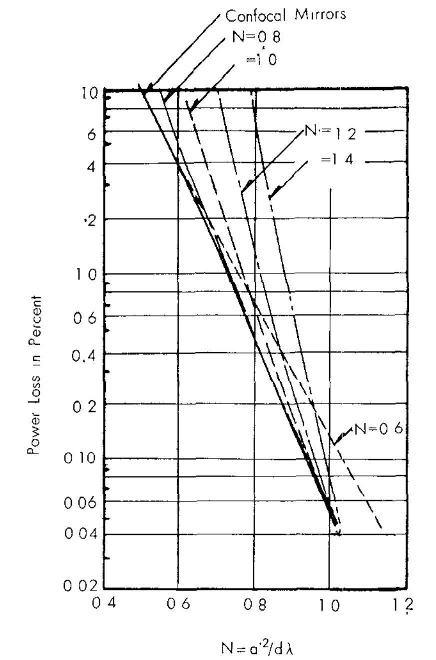

Fig. 1—Diffraction loss for a nonconfocal system of spherical mirrors.

On the other hand, it is sometimes required to calculate the variation of the diffraction loss for a nonconfocal system with the spacing between mirrors for constant mirror curvature and wavelength. In such a case, it seems more convenient to use the new parameter N' as defined by the formula

$$N' = \frac{a'^2}{d\lambda}.$$

With this parameter, the diffraction loss for a nonconfocal system can be illustrated for various values of N as shown in Fig. 1. The solid line in the figure denotes the loss curve for a confocal system for comparison. These curves seem to have sufficient accuracy in the range of $0.2b' < d < 1.8b'$.

When the diffraction loss for a nonconfocal system was measured for a variable mirror spacing, the measured value should be compared with the curve shown in Fig. 1, not with the loss curve for a confocal system. It is considered that the results of the measurement made by Beyer and Scheibe[5] may be compared more adequately with the curve in Fig. 1 for a given value of N.

ACKNOWLEDGMENT

The author wishes to thank Dr. F. Ikegami, the chief of the Second Radio Section in the Electrical Communications Laboratory for his support.

TAKEO OMORI
Electrical Communication Lab.
Nippon Telephone and Telegraph
Public Corp.
Tokyo, Japan

Manuscript received December 17, 1963.

[1] A. G. Fox and T. Li, "Resonant modes in a maser interferometer," *Bell Sys. Tech. J.*, vol. 40, pp. 453–488; March, 1961.

[2] G. Goubau and F. Schwering, "On the guided propagation of electromagnetic wave beams," IRE TRANS. ON ANTENNAS AND PROPAGATION, pp. 248–256; May, 1961.

[3] G. D. Boyd and J. P. Gordon, "Confocal multimode resonator for millimeter through optical wavelength masers," *Bell Sys. Tech. J.*, vol. 40, pp. 489–508; March, 1961.

[4] A. G. Fox and T. Li, "Modes in a maser interferometer with curved and tilted mirrors," PROC. IEEE, vol. 51, pp. 80–89; January, 1963.

[5] J. B. Beyer and E. H. Scheibe, "Loss measurements of the beam waveguide," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-11, pp. 18–22; January, 1963.

Semiconductor Switching and Limiting Using 3-db Short-Slot (Hybrid) Couplers

The silver-bonded germanium varactor diode has been successfully used as a switch and a limiter of microwave power when operated in a series mode between 9.0 and 9.6 Gc.[1] This report gives details of shunt mode switching and limiting using these same type diodes in conjunction with 3-db short-slot (hybrid) couplers. The technique of using 3-db short-slot (hybrid) couplers, but with other type diodes (*e.g.*, 1N263, MA-450, PIN's), has been reported by other investigators.[2–4]

Fig. 1 is a diagrammatic illustration of the short-slot (hybrid) coupler. If arms B and C are terminated in perfectly matched

Manuscript received December 19, 1963.

[1] V. J. Higgins, "*X*-band semiconductor switching and limiting using waveguide series tees," *Microwave J.*, vol. 6, pp. 77–83; November, 1963.

[2] R. Lucy, "Microwave High Speed Switch," *Proc. Natl. Electronics Components Conf.*, Philadelphia, Pa., pp. 12–15; May, 1959.

[3] R. V. Garver and D. Y. Tseng, "*X*-band diode limiting," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (*Correspondence*), vol. MTT-9, p. 202; March, 1961.

[4] W. F. Krupke, T. S. Hartwick and M. T. Weiss, "Solid-state *X*-band power limiter," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 472–480; November, 1961.

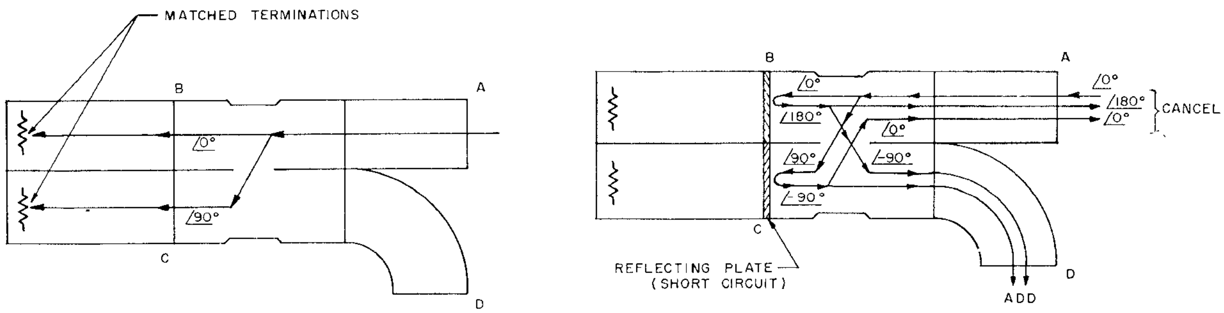

Fig. 1—(a) Diagrammatic view of 3-db short-slot (hybrid) coupler with balanced arms matched. (b) Diagrammatic view of 3-db short-slot (hybrid) coupler with balanced arms short circuited.

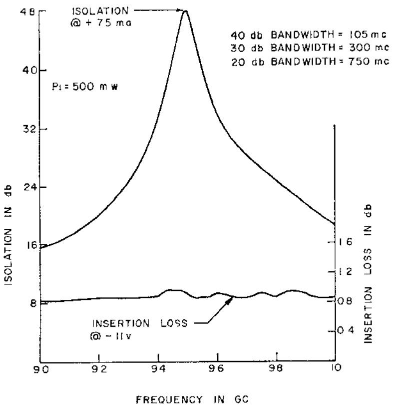

Fig. 2—Isolation and insertion loss vs frequency for 2-diode hybrid switch. Diodes fixed tuned with slide screw tuners for optimum performance at 9400 Mc.

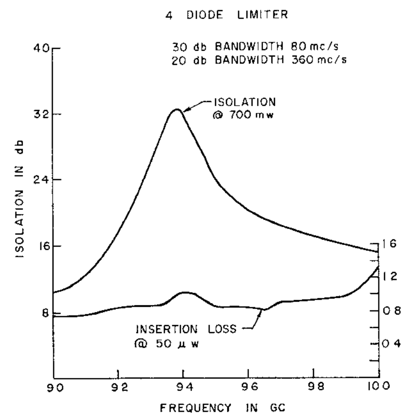

Fig. 3—Isolation and insertion loss vs frequency for 4-diode untuned limiter.

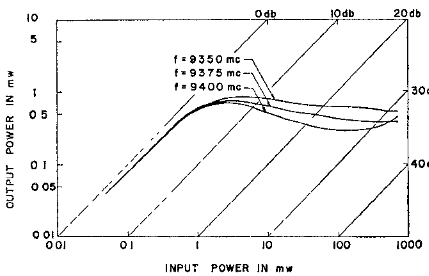

Fig. 4—Output characteristic of untuned limiter.

loads, as in Fig. 1(a), power entering arm *A* will divide evenly between arms *B* and *C* with the voltage entering arm *C* undergoing a 90° phase shift. The power in arms *B* and *C* will then be absorbed in the matched loads and the output arm (arm *D*) will be isolated from the input arm (arm *A*). When arms *B* and *C* are short circuited, power entering arm *A* again divides evenly between arms *B* and *C* with a 90° phase shift of the voltage in arm *C*. The voltages in arms *B* and *C* are now reflected with an additional 180° phase shift. Through hybrid action reflected voltages entering arm *A* destructively interfere with no reflected signal propagated in the input arm. Reflected voltages entering arm *D* constructively interfere and the total incoming signal in arm *A* leaves at arm *D*. This case is illustrated in Fig. 1(b) with incoming power arbitrarily taken to have 0° phase shift. Complete analysis of the 3-db short-slot (hybrid) coupler will be found in the literature.[5-7]

The simple *X*-band semiconductor switch consists of a diode mounted across a waveguide along the guide axis. The silver-bonded diode operated in this configuration will provide isolation when reverse biased and provide low insertion loss when forward biased.[1,3] Consider now two identical diode switches terminated in matched loads and mounted on each of the balanced arm of a 3-db short-slot (hybrid) coupler.

When the diodes are forward biased, most of the power incident upon the diodes will be transmitted to the matched loads with very little reflection seen in the output arm. In this bias condition the diodes provide isolation. With the diodes reverse biased most of the power incident upon the diodes is reflected and through hybrid action, as previously described, leaves through the output arm with little loss. In both cases the ratio of power in the input arm to power in the output arm defines isolation and insertion loss.

The silver-bonded diodes operated as hybrid switches at 500 mw CW have provided peak isolation of 32 to 35 db, with 20-db and 30-db isolation bandwidths of 825 Mc and 175 Mc, respectively. The use of slide screw tuners behind the diodes greatly enhances the switching performance. Fig. 2 shows the insertion loss and isolation as a function of frequency for a two-diode hybrid switch using slide screw tuners for better switching performance. The diodes were tuned for optimum performance at 9400 Mc.

With its biasing terminals short circuited, the hybrid switch functions well as a limiter. The operation of the diodes as a limiter is similar to that of the switch except that this operation is passive. Changes in diode impedance occur through changes in the incident power level. Thus, at incident power levels of a few hundred milliwatts the diodes present a high impedance across the line and most of the incident power is transmitted to the matched loads isolating the output arm of the hybrid. At incident power levels of $\frac{1}{2}$ mw and below, the diodes are a low impedance reflecting most of the incident power with little loss. Through hybrid action this reflected power leaves at the hybrid output arm. Hybrid limiters using two diodes and slide screw tuners have been made which provide peak isolation of 25 db with 20-db isolation bandwidth of 450 Mc and insertion loss of 0.9 db maximum over 800 Mc.

Better limiting has been obtained with an untuned, four-diode limiter, two diodes in each balanced arm spaced $5\lambda_g/4$ apart. The frequency dependence of this limiter is shown in Fig. 3 and the output characteristic in Fig. 4.

This report was an extension of investigations of the *X*-band switching capabilities of the silver-bonded germanium varactor diode. When used with short-slot hybrid couplers the reflection properties of the diode are utilized to provide relatively broad-band shunt mode switches and limiters in the 9- to 10-Gc region.

V. J. Higgins
U. S. Army Electronics Research
and Development Labs.
Fort Monmouth, N. J.

[5] M. J. Rodriguez and D. Weissman, "A microwave power limiter," IRE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-10, pp. 219–220; May, 1962.
[6] R. J. Mohr, "The design aspects of components utilizing symmetric 3 db hybrids," *Microwave J.*, vol. 5, pp. 2–6; June, 1962.
[7] H. J. Riblet, "Short slot hybrid junction," Proc. IRE. vol. 40, pp. 180–184; February, 1952.